

ENUMERATION OF SOME PARTICULAR SEXTUPLE PERSYMMETRIC MATRICES OVER \mathbb{F}_2 BY RANK

JORGEN CHERLY

RÉSUMÉ. Dans cet article nous comptons le nombre de certaines sextuples matrices persymétriques de rang i sur \mathbb{F}_2 .

ABSTRACT. In this paper we count the number of some particular sextuple persymmetric rank i matrices over \mathbb{F}_2 .

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1. INTRODUCTION

In this paper we propose to compute in the most simple case the number of sextuple persymmetric matrices with entries in \mathbb{F}_2 of rank i

That is to compute the number $\Gamma_i^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k}$ of sextuple persymmetric matrices in \mathbb{F}_2 of rank i ($0 \leq i \leq \inf(12, k)$) of the below form.

$$(1.1) \quad \left(\begin{array}{cccccc} \alpha_1^{(1)} & \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \dots & \alpha_k^{(1)} \\ \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \alpha_6^{(1)} & \dots & \alpha_{k+1}^{(1)} \\ \hline \alpha_1^{(2)} & \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \dots & \alpha_k^{(2)} \\ \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \alpha_6^{(2)} & \dots & \alpha_{k+1}^{(2)} \\ \hline \alpha_1^{(3)} & \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \dots & \alpha_k^{(3)} \\ \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \alpha_6^{(3)} & \dots & \alpha_{k+1}^{(3)} \\ \hline \alpha_1^{(4)} & \alpha_2^{(4)} & \alpha_3^{(4)} & \alpha_4^{(4)} & \alpha_5^{(4)} & \dots & \alpha_k^{(4)} \\ \alpha_2^{(4)} & \alpha_3^{(4)} & \alpha_4^{(4)} & \alpha_5^{(4)} & \alpha_6^{(4)} & \dots & \alpha_{k+1}^{(4)} \\ \hline \alpha_1^{(5)} & \alpha_2^{(5)} & \alpha_3^{(5)} & \alpha_4^{(5)} & \alpha_5^{(5)} & \dots & \alpha_k^{(5)} \\ \alpha_2^{(5)} & \alpha_3^{(5)} & \alpha_4^{(5)} & \alpha_5^{(5)} & \alpha_6^{(5)} & \dots & \alpha_{k+1}^{(5)} \\ \hline \alpha_1^{(6)} & \alpha_2^{(6)} & \alpha_3^{(6)} & \alpha_4^{(6)} & \alpha_5^{(6)} & \dots & \alpha_k^{(6)} \\ \alpha_2^{(6)} & \alpha_3^{(6)} & \alpha_4^{(6)} & \alpha_5^{(6)} & \alpha_6^{(6)} & \dots & \alpha_{k+1}^{(6)} \end{array} \right)$$

We remark that this paper is just a generalization of the results obtained in the author's paper [13] concerning quintuple persymmetric matrices in \mathbb{F}_2 .

2. NOTATIONS AND PRELIMINARIES

2.1. Some notations concerning the field of Laurent Series $\mathbb{F}_2((T^{-1}))$.

We denote by $\mathbb{F}_2((T^{-1})) = \mathbb{K}$ the completion of the field $\mathbb{F}_2(T)$, the field of rational functions over the finite field \mathbb{F}_2 , for the infinity valuation $\mathbf{v} = \mathbf{v}_\infty$ defined by $\mathbf{v}\left(\frac{A}{B}\right) = \deg B - \deg A$ for each pair (A,B) of non-zero polynomials. Then every element non-zero t in $\mathbb{F}_2\left(\left(\frac{1}{T}\right)\right)$ can be expanded in a unique way in a convergent Laurent series $t = \sum_{j=-\infty}^{-\mathbf{v}(t)} t_j T^j$ where $t_j \in \mathbb{F}_2$. We associate to the infinity valuation $\mathbf{v} = \mathbf{v}_\infty$ the absolute value $|\cdot|_\infty$ defined by

$$|t|_\infty = |t| = 2^{-\mathbf{v}(t)}.$$

We denote E the Character of the additive locally compact group $\mathbb{F}_2\left(\left(\frac{1}{T}\right)\right)$ defined by

$$E\left(\sum_{j=-\infty}^{-\mathbf{v}(t)} t_j T^j\right) = \begin{cases} 1 & \text{if } t_{-1} = 0, \\ -1 & \text{if } t_{-1} = 1. \end{cases}$$

We denote \mathbb{P} the valuation ideal in \mathbb{K} , also denoted the unit interval of \mathbb{K} , i.e. the open ball of radius 1 about 0 or, alternatively, the set of all Laurent series

$$\sum_{i \geq 1} \alpha_i T^{-i} \quad (\alpha_i \in \mathbb{F}_2)$$

and, for every rational integer j , we denote by \mathbb{P}_j the ideal $\{t \in \mathbb{K} \mid \mathbf{v}(t) > j\}$. The sets \mathbb{P}_j are compact subgroups of the additive locally compact group \mathbb{K} .

All $t \in \mathbb{F}_2\left(\left(\frac{1}{T}\right)\right)$ may be written in a unique way as $t = [t] + \{t\}$, $[t] \in \mathbb{F}_2[T]$, $\{t\} \in \mathbb{P}(=\mathbb{P}_0)$.

We denote by dt the Haar measure on \mathbb{K} chosen so that

$$\int_{\mathbb{P}} dt = 1.$$

$$\text{Let } (t_1, t_2, \dots, t_n) = \left(\sum_{j=-\infty}^{-\nu(t_1)} \alpha_j^{(1)} T^j, \sum_{j=-\infty}^{-\nu(t_2)} \alpha_j^{(2)} T^j, \dots, \sum_{j=-\infty}^{-\nu(t_n)} \alpha_j^{(n)} T^j \right) \in \mathbb{K}^n.$$

We denote ψ the Character on $(\mathbb{K}^n, +)$ defined by

$$\begin{aligned} \psi\left(\sum_{j=-\infty}^{-\nu(t_1)} \alpha_j^{(1)} T^j, \sum_{j=-\infty}^{-\nu(t_2)} \alpha_j^{(2)} T^j, \dots, \sum_{j=-\infty}^{-\nu(t_n)} \alpha_j^{(n)} T^j\right) &= E\left(\sum_{j=-\infty}^{-\nu(t_1)} \alpha_j^{(1)} T^j\right) \cdot E\left(\sum_{j=-\infty}^{-\nu(t_2)} \alpha_j^{(2)} T^j\right) \cdots E\left(\sum_{j=-\infty}^{-\nu(t_n)} \alpha_j^{(n)} T^j\right) \\ &= \begin{cases} 1 & \text{if } \alpha_{-1}^{(1)} + \alpha_{-1}^{(2)} + \dots + \alpha_{-1}^{(n)} = 0 \\ -1 & \text{if } \alpha_{-1}^{(1)} + \alpha_{-1}^{(2)} + \dots + \alpha_{-1}^{(n)} = 1 \end{cases} \end{aligned}$$

2.2. Some results concerning n-times persymmetric matrices over \mathbb{F}_2 .

$$\text{Set } (t_1, t_2, \dots, t_n) = \left(\sum_{i \geq 1} \alpha_i^{(1)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(2)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(3)} T^{-i}, \dots, \sum_{i \geq 1} \alpha_i^{(n)} T^{-i} \right) \in \mathbb{P}^n.$$

Denote by $D \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix} \times k (t_1, t_2, \dots, t_n)$

the following $2n \times k$ n-times persymmetric matrix over the finite field \mathbb{F}_2

$$(2.1) \quad \left(\begin{array}{cccccc} \alpha_1^{(1)} & \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \alpha_6^{(1)} & \dots & \alpha_k^{(1)} \\ \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \alpha_6^{(1)} & \alpha_7^{(1)} & \dots & \alpha_{k+1}^{(1)} \\ \hline \alpha_1^{(2)} & \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \alpha_6^{(2)} & \dots & \alpha_k^{(2)} \\ \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \alpha_6^{(2)} & \alpha_7^{(2)} & \dots & \alpha_{k+1}^{(2)} \\ \hline \alpha_1^{(3)} & \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \alpha_6^{(3)} & \dots & \alpha_k^{(3)} \\ \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \alpha_6^{(3)} & \alpha_7^{(3)} & \dots & \alpha_{k+1}^{(3)} \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline \alpha_1^{(n)} & \alpha_2^{(n)} & \alpha_3^{(n)} & \alpha_4^{(n)} & \alpha_5^{(n)} & \alpha_6^{(n)} & \dots & \alpha_k^{(n)} \\ \alpha_2^{(n)} & \alpha_3^{(n)} & \alpha_4^{(n)} & \alpha_5^{(n)} & \alpha_6^{(n)} & \alpha_7^{(n)} & \dots & \alpha_{k+1}^{(n)} \end{array} \right)$$

We denote by $\Gamma_i^{\left[\begin{smallmatrix} 2 \\ \vdots \\ 2 \end{smallmatrix} \right] \times k}$ the number of rank i n -times persymmetric matrices over \mathbb{F}_2 of the above form :

Let $f(t_1, t_2, \dots, t_n)$ be the exponential sum in \mathbb{P}^n defined by $(t_1, t_2, \dots, t_n) \in \mathbb{P}^n \longrightarrow$

$$\sum_{\deg Y \leq k-1} \sum_{\deg U_1 \leq 1} E(t_1 Y U_1) \sum_{\deg U_2 \leq 1} E(t_2 Y U_2) \dots \sum_{\deg U_n \leq 1} E(t_n Y U_n).$$

Then

$$f_k(t_1, t_2, \dots, t_n) = 2^{2n+k-\text{rank} \left[D \left[\begin{smallmatrix} 2 \\ \vdots \\ 2 \end{smallmatrix} \right] \right] \times k} (t_1, t_2, \dots, t_n)$$

Hence the number denoted by $R_{q,n}^{(k)}$ of solutions

$$(Y_1, U_1^{(1)}, U_2^{(1)}, \dots, U_n^{(1)}, Y_2, U_1^{(2)}, U_2^{(2)}, \dots, U_n^{(2)}, \dots, Y_q, U_1^{(q)}, U_2^{(q)}, \dots, U_n^{(q)}) \in (\mathbb{F}_2[T])^{(n+1)q}$$

of the polynomial equations

$$\begin{cases} Y_1 U_1^{(1)} + Y_2 U_1^{(2)} + \dots + Y_q U_1^{(q)} = 0 \\ Y_1 U_2^{(1)} + Y_2 U_2^{(2)} + \dots + Y_q U_2^{(q)} = 0 \\ \vdots \\ Y_1 U_n^{(1)} + Y_2 U_n^{(2)} + \dots + Y_q U_n^{(q)} = 0 \end{cases}$$

$$\Leftrightarrow \begin{pmatrix} U_1^{(1)} & U_1^{(2)} & \cdots & U_1^{(q)} \\ U_2^{(1)} & U_2^{(2)} & \cdots & U_2^{(q)} \\ \vdots & \vdots & \vdots & \vdots \\ U_n^{(1)} & U_n^{(2)} & \cdots & U_n^{(q)} \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

satisfying the degree conditions

$$\deg Y_i \leq k-1, \quad \deg U_j^{(i)} \leq 1, \quad \text{for } 1 \leq j \leq n \quad 1 \leq i \leq q$$

is equal to the following integral over the unit interval in \mathbb{K}^n

$$\int_{\mathbb{P}^n} f_k^q(t_1, t_2, \dots, t_n) dt_1 dt_2 \dots dt_n.$$

Observing that $f(t_1, t_2, \dots, t_n)$ is constant on cosets of $\prod_{j=1}^n \mathbb{P}_{k+1}$ in \mathbb{P}^n the above integral is equal to

$$(2.2) \quad 2^{q(2n+k)-(k+1)n} \sum_{i=0}^{\inf(2n,k)} \Gamma_i^{\begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix} \times k} 2^{-iq} = R_{q,n}^{(k)}$$

Recall that $R_{q,n}^{(k)}$ is equal to the number of solutions of the polynomial system

$$(2.3) \quad \begin{pmatrix} U_1^{(1)} & U_1^{(2)} & \cdots & U_1^{(q)} \\ U_2^{(1)} & U_2^{(2)} & \cdots & U_2^{(q)} \\ \vdots & \vdots & \vdots & \vdots \\ U_n^{(1)} & U_n^{(2)} & \cdots & U_n^{(q)} \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

satisfying the degree conditions

$$\deg Y_i \leq k-1, \quad \deg U_j^{(i)} \leq 1, \quad \text{for } 1 \leq j \leq n \quad 1 \leq i \leq q$$

From (2.2) we obtain for $q = 1$

$$(2.4) \quad 2^{k-(k-1)n} \sum_{i=0}^{\inf(2n,k)} \Gamma_i^{\begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix} \times k} 2^{-i} = R_{1,n}^{(k)} = 2^{2n} + 2^k - 1$$

We have obviously

$$(2.5) \quad \sum_{i=0}^k \Gamma_i^{\begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix} \times k} = 2^{(k+1)n}$$

From the fact that the number of rank one persymmetric matrices over \mathbb{F}_2 is equal to three we obtain using combinatorial methods :

$$(2.6) \quad \Gamma_1 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix} \times k = (2^n - 1) \cdot 3$$

For more details see Cherly [11,12].

2.3. The case n=6.

Set $(t_1, t_2, t_3, t_4, t_5, t_6)$

$$= \left(\sum_{i \geq 1} \alpha_i^{(1)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(2)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(3)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(4)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(5)} T^{-i}, \sum_{i \geq 1} \alpha_i^{(6)} T^{-i} \right) \in \mathbb{P}^6.$$

Denote by $D \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k (t_1, t_2, t_3, t_4, t_5, t_6)$

the following $12 \times k$ sextuple persymmetric matrix over the finite field \mathbb{F}_2

$$\begin{pmatrix} \alpha_1^{(1)} & \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \dots & \alpha_k^{(1)} \\ \alpha_2^{(1)} & \alpha_3^{(1)} & \alpha_4^{(1)} & \alpha_5^{(1)} & \alpha_6^{(1)} & \dots & \alpha_{k+1}^{(1)} \\ \hline \alpha_1^{(2)} & \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \dots & \alpha_k^{(2)} \\ \alpha_2^{(2)} & \alpha_3^{(2)} & \alpha_4^{(2)} & \alpha_5^{(2)} & \alpha_6^{(2)} & \dots & \alpha_{k+1}^{(2)} \\ \hline \alpha_1^{(3)} & \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \dots & \alpha_k^{(3)} \\ \alpha_2^{(3)} & \alpha_3^{(3)} & \alpha_4^{(3)} & \alpha_5^{(3)} & \alpha_6^{(3)} & \dots & \alpha_{k+1}^{(3)} \\ \hline \alpha_1^{(4)} & \alpha_2^{(4)} & \alpha_3^{(4)} & \alpha_4^{(4)} & \alpha_5^{(4)} & \dots & \alpha_k^{(4)} \\ \alpha_2^{(4)} & \alpha_3^{(4)} & \alpha_4^{(4)} & \alpha_5^{(4)} & \alpha_6^{(4)} & \dots & \alpha_{k+1}^{(4)} \\ \hline \alpha_1^{(5)} & \alpha_2^{(5)} & \alpha_3^{(5)} & \alpha_4^{(5)} & \alpha_5^{(5)} & \dots & \alpha_k^{(5)} \\ \alpha_2^{(5)} & \alpha_3^{(5)} & \alpha_4^{(5)} & \alpha_5^{(5)} & \alpha_6^{(5)} & \dots & \alpha_{k+1}^{(5)} \\ \hline \alpha_1^{(6)} & \alpha_2^{(6)} & \alpha_3^{(6)} & \alpha_4^{(6)} & \alpha_5^{(6)} & \dots & \alpha_k^{(6)} \\ \alpha_2^{(6)} & \alpha_3^{(6)} & \alpha_4^{(6)} & \alpha_5^{(6)} & \alpha_6^{(6)} & \dots & \alpha_{k+1}^{(6)} \end{pmatrix}$$

We denote by $\Gamma_i \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k$ the number of rank i sextuple persymmetric matrices over \mathbb{F}_2 of the above form :

$$\text{Let } f(t_1, t_2, t_3, t_4, t_5, t_6) \text{ be the exponential sum in } \mathbb{P}^6 \text{ defined by} \\ (t_1, t_2, t_3, t_4, t_5, t_6) \in \mathbb{P}^6 \longrightarrow \sum_{\deg Y \leq k-1} \sum_{\deg U_1 \leq 1} E(t_1 Y U_1) \sum_{\deg U_2 \leq 1} E(t_2 Y U_2) \sum_{\deg U_3 \leq 1} E(t_3 Y U_3)$$

$$\sum_{\deg U_4 \leq 1} E(t_4 Y U_4) \sum_{\deg U_5 \leq 1} E(t_5 Y U_5) \sum_{\deg U_6 \leq 1} E(t_6 Y U_6).$$

Then

$$f_k(t_1, t_2, t_3, t_4, t_5, t_6) = 2^{12+k-\text{rank}} \left[D \begin{bmatrix} 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 2 \end{bmatrix} \times k \right]_{(t_1, t_2, t_3, t_4, t_5, t_6)}$$

Hence the number denoted by $R_{q,6}^{(k)}$ of solutions

$$(Y_1, U_1^{(1)}, U_2^{(1)}, U_3^{(1)}, U_4^{(1)}, U_5^{(1)}, U_6^{(1)}, Y_2, U_1^{(2)}, U_2^{(2)}, U_3^{(2)}, U_4^{(2)}, U_5^{(2)}, U_6^{(2)} \dots Y_q, U_1^{(q)}, U_2^{(q)}, U_3^{(q)}, U_4^{(q)}, U_5^{(q)}, U_6^{(q)}) \in (\mathbb{F}_2[T])^{7q}$$

of the polynomial equations

$$\begin{cases} Y_1 U_1^{(1)} + Y_2 U_1^{(2)} + \dots + Y_q U_1^{(q)} = 0 \\ Y_1 U_2^{(1)} + Y_2 U_2^{(2)} + \dots + Y_q U_2^{(q)} = 0 \\ Y_1 U_3^{(1)} + Y_2 U_3^{(2)} + \dots + Y_q U_3^{(q)} = 0 \\ Y_1 U_4^{(1)} + Y_2 U_4^{(2)} + \dots + Y_q U_4^{(q)} = 0 \\ Y_1 U_5^{(1)} + Y_2 U_5^{(2)} + \dots + Y_q U_5^{(q)} = 0 \\ Y_1 U_6^{(1)} + Y_2 U_6^{(2)} + \dots + Y_q U_6^{(q)} = 0 \end{cases}$$

$$\Leftrightarrow \begin{pmatrix} U_1^{(1)} & U_1^{(2)} & \dots & U_1^{(q)} \\ U_2^{(1)} & U_2^{(2)} & \dots & U_2^{(q)} \\ U_3^{(1)} & U_3^{(2)} & \dots & U_3^{(q)} \\ U_4^{(1)} & U_4^{(2)} & \dots & U_4^{(q)} \\ U_5^{(1)} & U_5^{(2)} & \dots & U_5^{(q)} \\ U_6^{(1)} & U_6^{(2)} & \dots & U_6^{(q)} \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

satisfying the degree conditions

$$\deg Y_i \leq k-1, \quad \deg U_j^{(i)} \leq 1, \quad \text{for } 1 \leq j \leq 6 \quad 1 \leq i \leq q$$

is equal to the following integral over the unit interval in \mathbb{K}^6

$$\int_{\mathbb{P}^6} f_k^q(t_1, t_2, t_3, t_4, t_5, t_6) dt_1 dt_2 dt_3 dt_4 dt_5 dt_6$$

Observing that $f(t_1, t_2, t_3, t_4, t_5, t_6)$ is constant on cosets of $\prod_{j=1}^6 \mathbb{P}_{k+1}$ in \mathbb{P}^6 the above integral is equal to

$$(2.7) \quad 2^{q(12+k)-6(k+1)} \sum_{i=0}^{\inf(12,k)} \Gamma_i \begin{bmatrix} 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 2 \end{bmatrix} \times k \quad 2^{-iq} = R_{q,6}^{(k)} \quad \text{where } k \geq 1$$

2.4. Some preliminary results.

Lemma 2.1.

$$(2.8) \quad \left\{ \begin{array}{l} \Gamma_0 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = 1 \quad \text{if } k \geq 1 \\ \Gamma_1 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = 3 \cdot (2^n - 1) \quad \text{if } k \geq 2 \\ \Gamma_2 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = (2^{n+1} - 2) \cdot 2^k + 7 \cdot 2^{2n} - 25 \cdot 2^n + 18 \quad \text{for } k \geq 3 \\ \Gamma_3 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = [7 \cdot 2^{2n} - 21 \cdot 2^n + 14] \cdot 2^k + 15 \cdot 2^{3n} - 133 \cdot 2^{2n} + 294 \cdot 2^n - 176 \quad \text{for } k \geq 4 \\ \Gamma_4 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = \frac{1}{3} \cdot (2^{2n+1} - 6 \cdot 2^n + 4) \cdot 2^{2k} \\ \quad + \frac{1}{6} \cdot (105 \cdot 2^{3n} - 783 \cdot 2^{2n} + 1614 \cdot 2^n - 936) \cdot 2^k \\ \quad + \frac{1}{6} \cdot (186 \cdot 2^{4n} - 3630 \cdot 2^{3n} + 19028 \cdot 2^{2n} - 34464 \cdot 2^n + 18880) \quad \text{for } k \geq 5 \\ \Gamma_5 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = \frac{1}{2} \cdot (5 \cdot 2^{3n} - 35 \cdot 2^{2n} + 70 \cdot 2^n - 40) \cdot 2^{2k} \\ \quad + \frac{1}{4} \cdot (155 \cdot 2^{4n} - 2565 \cdot 2^{3n} + 12530 \cdot 2^{2n} - 21960 \cdot 2^n + 11840) \cdot 2^k \\ \quad + 63 \cdot 2^{5n} - 2573 \cdot 2^{4n} + 29150 \cdot 2^{3n} - 123760 \cdot 2^{2n} + 203872 \cdot 2^n - 106752 \quad \text{for } k \geq 6 \\ \Gamma_6 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = \frac{1}{21} \cdot (2^{3n} - 7 \cdot 2^{2n} + 14 \cdot 2^n - 8) \cdot 2^{3k} \\ \quad + \frac{1}{168} \cdot (1085 \cdot 2^{4n} - 16723 \cdot 2^{3n} + 79086 \cdot 2^{2n} - 136472 \cdot 2^n + 73024) \cdot 2^{2k} \\ \quad + \frac{1}{168} \cdot (13671 \cdot 2^{5n} - 475881 \cdot 2^{4n} + 5026378 \cdot 2^{3n} - 20647816 \cdot 2^{2n} + 33473216 \cdot 2^n - 17389568) \cdot 2^k \\ \quad + \frac{1}{168} \cdot (21336 \cdot 2^{6n} - 1781640 \cdot 2^{5n} + 41896624 \cdot 2^{4n} \\ \quad - 382091648 \cdot 2^{3n} + 1470524160 \cdot 2^{2n} - 2311493632 \cdot 2^n + 1182924800) \quad \text{for } k \geq 7 \\ \Gamma_7 \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = \frac{31}{168} \cdot [2^{4n} - 15 \cdot 2^{3n} + 70 \cdot 2^{2n} - 120 \cdot 2^n + 64] \cdot 2^{3k} \\ \quad + \frac{1}{96} \cdot [1395 \cdot 2^{5n} - 45229 \cdot 2^{4n} + 462210 \cdot 2^{3n} - 1868680 \cdot 2^{2n} + 3005760 \cdot 2^n - 1555456] \cdot 2^{2k} \\ \quad + \frac{1}{48} \cdot [8001 \cdot 2^{6n} - 571023 \cdot 2^{5n} + 12524806 \cdot 2^{4n} - 110524920 \cdot 2^{3n} + 418606144 \cdot 2^{2n} \\ \quad - 652818432 \cdot 2^n + 332775424] \cdot 2^k \\ \quad + \frac{1}{21} \cdot [5355 \cdot 2^{7n} - 904113 \cdot 2^{6n} + 43302294 \cdot 2^{5n} - 817168432 \cdot 2^{4n} + 6743660640 \cdot 2^{3n} \\ \quad - 96649567 \cdot 2^8 \cdot 2^{2n} + 4637778 \cdot 2^{13} \cdot 2^n - 293263 \cdot 2^{16}] \quad \text{for } k \geq 8 \end{array} \right.$$

Proof. We recall the result obtained in Lemma 3.3 [15] concerning the number of rank i n -times persymmetric matrices over \mathbb{F}_2 of the form (2.1) for $0 \leq i \leq 7$. \square

Lemma 2.2.

(2.9)

$$\begin{cases} \sum_{i=0}^{\inf(2n,k)} \Gamma_i \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = 2^{(k+1)n}, \\ \sum_{i=0}^{\inf(2n,k)} \Gamma_i \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} 2^{-i} = 2^{n+k(n-1)} + 2^{(k-1)n} - 2^{(k-1)n-k}, \\ \sum_{i=0}^{\inf(2n,k)} \Gamma_i \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} 2^{-2i} = 2^{n+k(n-2)} + 2^{-n+k(n-2)} \cdot [3 \cdot 2^k - 3] + 2^{-2n+k(n-2)} \cdot [6 \cdot 2^{k-1} - 6] \\ + 2^{-3n+kn} - 6 \cdot 2^{n(k-3)-k} + 8 \cdot 2^{-3n+k(n-2)}. \end{cases}$$

Proof. Recall the relations (3.15) in [15]. \square

Lemma 2.3. *The number of rank $2n$ n -times persymmetric matrices of the form (2.1) is equal to :*

$$(2.10) \quad \Gamma_{2n} \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = 2^n \prod_{j=1}^n (2^k - 2^{2n-j})$$

Proof. Use the formula (2.1) in [10] with $s_1 = s_2 = \dots = s_m = 2$, $\delta = \sum_{j=1}^m s_j = 2n$ and $m=n$. \square

Lemma 2.4. *We postulate that the number of sextuple persymmetric matrices of the form (1.1) of rank i can be expressed in the following manner :*

(2.11)

$$\Gamma_i \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times k} = \begin{cases} 1 & \text{if } i = 0, \\ a_1 & \text{if } i = 1, \\ a_2 \cdot 2^k + b_2 & \text{if } i = 2, \\ a_3 \cdot 2^k + b_3 & \text{if } i = 3, \\ a_4 \cdot 2^{2k} + b_4 \cdot 2^k + c_4 & \text{if } i = 4, \\ a_5 \cdot 2^{2k} + b_5 \cdot 2^k + c_5 & \text{if } i = 5, \\ a_6 \cdot 2^{3k} + b_6 \cdot 2^{2k} + c_6 \cdot 2^k + d_6 & \text{if } i = 6, \\ a_7 \cdot 2^{3k} + b_7 \cdot 2^{2k} + c_7 \cdot 2^k + d_7 & \text{if } i = 7, \\ a_8 \cdot 2^{4k} + b_8 \cdot 2^{3k} + c_8 \cdot 2^{2k} + d_8 \cdot 2^k + e_8 & \text{if } i = 8, \\ a_9 \cdot 2^{4k} + b_9 \cdot 2^{3k} + c_9 \cdot 2^{2k} + d_9 \cdot 2^k + e_9 & \text{if } i = 9, \\ a_{10} \cdot 2^{5k} + b_{10} \cdot 2^{4k} + c_{10} \cdot 2^{3k} + d_{10} \cdot 2^{2k} + e_{10} \cdot 2^k + f_{10} & \text{if } i = 10, \\ a_{11} \cdot 2^{5k} + b_{11} \cdot 2^{4k} + c_{11} \cdot 2^{3k} + d_{11} \cdot 2^{2k} + e_{11} \cdot 2^k + f_{11} & \text{if } i = 11, \\ a_{12} \cdot 2^{6k} + b_{12} \cdot 2^{5k} + c_{12} \cdot 2^{4k} + d_{12} \cdot 2^{3k} + e_{12} \cdot 2^{2k} + f_{12} \cdot 2^k + g_{12} & \text{if } i = 12. \end{cases}$$

Proof. To justify our assumption we recall that in the case $n=2$ (see [5]) we have :

$$\Gamma_i^{\left[\begin{smallmatrix} 2 \\ 2 \end{smallmatrix}\right] \times k} = \begin{cases} 1 & \text{if } i = 0, k \geq 1, \\ 9 & \text{if } i = 1, k > 1, \\ 6 \cdot 2^k + 30 & \text{if } i = 2, k > 2, \\ 42 \cdot 2^k - 168 & \text{if } i = 3, k > 3, \\ 4 \cdot 2^{2k} - 48 \cdot 2^k + 128 & \text{if } i = 4, k \geq 4 \end{cases}$$

and in the case $n=3$ (see [6]) :

$$\Gamma_i^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = \begin{cases} 1 & \text{if } i = 0 \\ 21 & \text{if } i = 1 \\ 14 \cdot 2^k + 266 & \text{if } i = 2 \\ 294 \cdot 2^k + 1344 & \text{if } i = 3 \\ 28 \cdot 2^{2k} + 2604 \cdot 2^k - 22624 & \text{if } i = 4 \\ 420 \cdot 2^{2k} - 10080 \cdot 2^k + 53760 & \text{if } i = 5 \\ 8 \cdot 2^{3k} - 448 \cdot 2^{2k} + 7168 \cdot 2^k - 32768 & \text{if } i = 6, k \geq 6 \end{cases}$$

Recall similar expressions concerning quadruple, quintuple persymmetric matrices see [12,13]. \square

Lemma 2.5. *We postulate :*

$$(2.12) \quad \left\{ \begin{array}{l} \Gamma_7^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k = 6 \\ \Gamma_8^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k \in \{6, 7\} \\ \Gamma_9^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k \in \{6, 7, 8\} \\ \Gamma_{10}^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k \in \{6, 7, 8, 9\} \\ \Gamma_{11}^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k \in \{6, 7, 8, 9, 10\} \\ \Gamma_{12}^{\left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix}\right] \times k} = 0 \quad \text{for } k \in \{6, 7, 8, 9, 10, 11\} \end{array} \right.$$

That is :

$$\left\{ \begin{array}{l} \Gamma_7 \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = (2^k - 2^6) \cdot (\alpha \cdot 2^{2k} + \dots) \\ \Gamma_8 \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = (2^k - 2^6)(2^k - 2^7) \cdot (\alpha \cdot 2^{2k} + \dots) \\ \Gamma_9 \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = (2^k - 2^6)(2^k - 2^7)(2^k - 2^8) \cdot (\alpha \cdot 2^k + \dots) \\ \Gamma_{10} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = (2^k - 2^6)(2^k - 2^7)(2^k - 2^8)(2^k - 2^9) \cdot (\alpha \cdot 2^k + \dots) \\ \Gamma_{11} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = \alpha(2^k - 2^6)(2^k - 2^7)(2^k - 2^8)(2^k - 2^9)(2^k - 2^{10}) \\ \Gamma_{12} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = \alpha(2^k - 2^6)(2^k - 2^7)(2^k - 2^8)(2^k - 2^9)(2^k - 2^{10})(2^k - 2^{11}) \end{array} \right.$$

Proof. To justify our assumption we have in the case n=2 :

$$\Gamma_i^{\begin{bmatrix} 2 \\ 2 \end{bmatrix} \times k} = \begin{cases} 1 & \text{if } i = 0, k \geq 1, \\ 9 & \text{if } i = 1, k > 1, \\ 6 \cdot 2^k + 30 & \text{if } i = 2, k > 2, \\ 42 \cdot 2^k - 168 = 42 \cdot (2^k - 2^2) & \text{if } i = 3, k > 3, \\ 4 \cdot 2^{2k} - 48 \cdot 2^k + 128 = 4 \cdot (2^k - 2^2)(2^k - 2^3) & \text{if } i = 4, k \geq 4 \end{cases}$$

and in the case n=3 :

$$\Gamma_i^{\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \times k} = \begin{cases} 1 & \text{if } i = 0 \\ 21 & \text{if } i = 1 \\ 14 \cdot 2^k + 266 & \text{if } i = 2 \\ 294 \cdot 2^k + 1344 & \text{if } i = 3 \\ 28 \cdot 2^{2k} + 2604 \cdot 2^k - 22624 = (2^k - 2^3)(28 \cdot 2^k + 2828) & \text{if } i = 4 \\ 420 \cdot 2^{2k} - 10080 \cdot 2^k + 53760 = 420 \cdot (2^k - 2^3)(2^k - 2^4) & \text{if } i = 5 \\ 8 \cdot 2^{3k} - 448 \cdot 2^{2k} + 7168 \cdot 2^k - 32768 = 8 \cdot (2^k - 2^3)(2^k - 2^4)(2^k - 2^5) & \text{if } i = 6, k \geq 6 \end{cases}$$

See also the similar problem concerning quintuple persymmetric matrices [13]. \square

Lemma 2.6.

$$(2.13) \quad \left\{ \begin{array}{l} \sum_{i=0}^{12} \Gamma_i \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k = 2^{6k+6}, \\ \sum_{i=0}^{12} \Gamma_i \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k \quad 2^{12-i} = 2^{6k+6} + 262080 \cdot 2^{5k}, \\ \sum_{i=0}^{12} \Gamma_i \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \times k \quad 2^{24-2i} = 2^{6k+6} + 798336 \cdot 2^{5k} + 1072931328 \cdot 2^{4k} \end{array} \right.$$

Proof. Apply (2.9) with $n=6$. □

2.5. Computation of the number of sextuple persymmetric matrices of the form (1.1) of rank I.

(2.14)

$$(2.14) \quad \left\{ \begin{array}{l} \Gamma_0 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 1 \quad \text{if } k \geq 1 \\ \Gamma_1 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 189 \quad \text{if } k \geq 2 \\ \Gamma_2 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 126 \cdot 2^k + 27090 \quad \text{for } k \geq 3 \\ \Gamma_3 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 27342 \cdot 2^k + 3406032 \quad \text{for } k \geq 4 \\ \Gamma_4 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 2604 \cdot 2^{2k} + 4070052 \cdot 2^k + 374121888 \quad \text{for } k \geq 5 \\ \Gamma_5 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 585900 \cdot 2^{2k} + 494499600 \cdot 2^k + 123537015 \cdot 2^8 \quad \text{for } k \geq 6 \\ \Gamma_6 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 11160 \cdot 2^{3k} + 84135240 \cdot 2^{2k} + 2^8 \cdot 184392495 \cdot 2^k + 29391255 \cdot 2^{15} \quad \text{for } k \geq 7 \\ \Gamma_7 \left[\begin{smallmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{smallmatrix} \right] \times k = 2421720 \cdot 2^{3k} + 277589655 \cdot 2^5 \cdot 2^{2k} + 2431729125 \cdot 2^{10} \cdot 2^k \\ - 2996595315 \cdot 2^{16} \quad \text{for } k \geq 8 \end{array} \right.$$

$$(2.15) \quad \left\{ \begin{array}{l} \Gamma_8 \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times k = 10416 \cdot 2^{4k} + 216944 \cdot 1395 \cdot 2^{3k} + 2155757205 \cdot 2^8 \cdot 2^{2k} \\ \quad - 6999385995 \cdot 2^{14} \cdot 2^k + 4767802914 \cdot 2^{20} \quad \text{for } k \geq 9 \\ \Gamma_9 \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times k = 1968624 \cdot 2^{4k} + 15196608 \cdot 1395 \cdot 2^{3k} - 2387571795 \cdot 2^{12} \cdot 2^{2k} \\ \quad + 4814516070 \cdot 2^{18} \cdot 2^k - 2760151464 \cdot 2^{24} \quad \text{for } k \geq 10 \\ \Gamma_{10} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times k = 2016 \cdot [2^{5k} + 81685 \cdot 2^{4k} - 79052480 \cdot 2^{3k} + 2^{13} \cdot 2888735 \cdot 2^{2k} \\ \quad - 1239163 \cdot 2^{21} \cdot 2^k + 2^{30} \cdot 82645] \quad \text{for } k \geq 11 \\ \Gamma_{11} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times k = 256032 \cdot [2^{5k} - 1984 \cdot 2^{4k} \\ \quad + 1269760 \cdot 2^{3k} - 325058560 \cdot 2^{2k} + 31744 \cdot 2^{20} \cdot 2^k - 2^{40}] \quad \text{for } k \geq 12 \\ \Gamma_{12} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times k = 2^6 \cdot [2^{6k} - 63 \cdot 2^6 \cdot 2^{5k} + 651 \cdot 2^{13} \cdot 2^{4k} - 1395 \cdot 2^{21} \cdot 2^{3k} \\ \quad + 651 \cdot 2^{30} \cdot 2^{2k} - 63 \cdot 2^{40} \cdot 2^k + 2^{51}] \quad \text{for } k \geq 12 \end{array} \right.$$

Proof. The proof is just a generalization of the similar proof of **Theorem 2.1** in [13]

We proceed as follows :

To prove (2.14) we apply (2.8) with $n=6$.

To prove (2.15) we combine (2.10) with $n=6$, (2.11), (2.12) and (2.13). \square

Example. Computation of $\Gamma_i \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times 6$ for $0 \leq i \leq 6$

$$\Gamma_i \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] \times 6 = \begin{cases} 1 & \text{for } i = 0 \\ 189 & \text{for } i = 1 \\ 35154 & \text{for } i = 2 \\ 5155920 & \text{for } i = 3 \\ 645271200 & \text{for } i = 4 \\ 256536315 \cdot 2^8 & \text{for } i = 5 \\ 2^{14} \cdot 264387375 & \text{for } i = 6 \end{cases}$$

Proof. Apply (2.14) with $k=6$. \square

Computing $\Gamma_i \begin{bmatrix} 2 \\ \vdots \\ 2 \end{bmatrix}^{\times 6}$ for $n=6$ we obtain the same result (see [14]).

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DÉPARTEMENT DE MATHÉMATIQUES, UNIVERSITÉ DE BREST, 29238 BREST CEDEX 3,
FRANCE

E-mail address: `Jorgen.Cherly@univ-brest.fr`

E-mail address: `andersen69@wanadoo.fr`